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Specifying a Structural Matching Game of Trading Networks with Transferable Utility[†]

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Matching games model relationship formation in a market setting. In transferable utility matching games, matched agents exchange monetary transfers or prices. There is increasing interest in the structural estimation of matching games with transferable utility in many fields of economics including development, family, finance, industrial organization, labor, marketing, and strategy (e.g., Ahlin 2016; Akkus, Cookson, and Hortaçsu 2016; Baccara et al. 2012; Chiappori and Salanié 2016; Chiappori, Salanié, and Weiss 2016; Chen and Song 2013; Choo 2015; Choo and Siow 2006; Fox 2010, 2016; Fox, Yang, and Hsu forthcoming; Galichon and Salanié 2015; Graham 2011; Mindruta, Moeen, and Agarwal 2016; Siow 2015; Yang, Shi, and Goldfarb 2009). This paper explores matching games of *trading networks* where agents can make more than one match and where agents are not necessarily a priori divided into two sides, as in men and women.

This article lays out many modeling decisions that need to be made when structurally estimating matching games. These are issues that are germane to prior studies but are not discussed there in detail for the sake of conciseness. I do not discuss particular estimators or inference procedures.

I. Matching with Trading Networks

Models of matching with trading networks use fairly flexible notation (Hatfield et al. 2013). Let a matching market be indexed by *agents* $i \in I$. Let there be a *space of trades* Ω. A *trade* $\omega \in \Omega$ includes the identity of the *buyer* and *seller*, $b(\omega) \in I$ and $s(\omega) \in I$. In some models, trades ω index other features, such as the hours of weekly work in a labor market. Each trade $\omega \in \Omega$ has an equilibrium price p_ω.

An agent $i \in I$ who buys trades Φ and sells trades Ψ at the prices p^Ω has a *profit* of

$$v^i(\Phi, \Psi) - \sum_{\omega \in \Phi} p_\omega + \sum_{\omega \in \Psi} p_\omega,$$

where $v^i(\Phi, \Psi)$ is the *valuation* of agent i for the buyer and seller trades. The fact that the prices enter additively is why this model has transferable utility; $v^i(\Phi, \Psi)$ is expressed in monetary units before any scale normalization in estimation. The valuation from making no trades is normalized to zero.

A *market outcome* is sets of trades $(\Phi_i, \Psi_i)_{i \in I}$ for all agents and a vector of the prices of all trades $(p_\omega)_{\omega \in \Omega}$. In the structural estimation of a matching game, the dependent variable is some aspect of the market outcome and the independent variables are some aspects of the spaces of agents I and of trades Ω. The estimation object is some aspect of the valuations of agents $v^i(\Phi, \Psi)$.

There are many types of matching games nested within the structure of matching with trading networks. In *one-to-one matching*, agents make only a single match; this can be encompassed by setting valuations to $-\infty$ if the number of elements of $\Phi_i \cup \Psi_i$ is more than 1. In *two-sided matching*, an agent is specified to be a buyer or seller ex ante; $v^i(\Phi_i, \Psi_i) = -\infty$ if a buyer agent sells trades or a seller agent buys trades. The notation also encompasses *many-to-many matching*, where an agent can buy and sell multiple trades as part of the same market outcome.

An econometric model must specify which variables are observed by the agents in the matching game but unobserved by the econometrician and what variables are observed by the

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agents and also measured in the data. Note that most of the papers cited above impose symmetric information: there are no variables observed by one agent in the game but not another agent.

Decompose agent i's valuation for trades (Φ, Ψ) as

$$v^i(\Phi, \Psi) = \pi(x_i, \Phi, \Psi) + \epsilon^i_{\Phi, \Psi},$$

where $\pi(x_i, \Phi, \Psi)$ is a function of *observables to the researcher*, including a vector of i's characteristics x_i, and $\epsilon^i_{\Phi,\Psi}$ is *unobserved heterogeneity* in i's valuation. If $\pi(x_i, \Phi, \Psi)$ is known up to a finite vector of parameters θ, as in $\pi(x_i, \Phi, \Psi) = X(x_i, \Phi, \Psi)'\theta$ for a vector of observables $X(x_i, \Phi, \Psi)$, then this function is parametrically specified; otherwise it is *nonparametric*. Likewise, if the distribution of the vector $(\epsilon^i_{\Phi,\Psi})^{i \in I}_{\Phi \subseteq \Omega, \Psi \subseteq \Omega}$ is known to the researcher after imposing location and scale normalizations, as in Choo and Siow (2006), or is known up to a finite vector of parameters, as in Galichon and Salanié (2015), this distribution is parametric.

II. Specifying the Structural Matching Game

A. *Cooperative versus Noncooperative Games*

A well specified matching game needs a *solution concept*. Matching games can in principle be analyzed under both *cooperative* game theoretic solution concepts and *noncooperative* solution concepts. One cooperative solution concept is *pairwise stability*: a market outcome is pairwise stable if there exists no vector of prices for trades $(p_\omega)_{\omega \in \Omega}$ such that two agents would prefer to make a trade ω instead of or in addition to their trades in the market outcome and (typically) no agent would prefer to drop trades. One noncooperative solution concept would be to specify an *auction* of complete or incomplete information, for example where sellers submit sealed bids to buyers for each buying opportunity and buyers pick the lowest bidder for each buying opportunity if that bid is lower than a reserve price.

The empirical papers cited previously use cooperative solution concepts while some structural models of say mergers in industrial organization use noncooperative concepts (e.g., Gowrisankaran 1999; Jeziorski 2014; Perez-Saiz 2015). Noncooperative game theory easily allows externalities from say post-merger competition between non-merging firms and the modeling of asymmetric information. Basing structural estimation off of any solution concept will lead to inconsistency if the solution concept is misspecified. However, casual intuition suggests that noncooperative solution concepts are often overly specific for particular decentralized markets and so are particularly likely to be misspecified. One researcher might model mergers as a first-price auction of complete information, another a first-price auction of incomplete information, another a second-price auction, another as some sort of bargaining model, and so forth. Meanwhile, it is possible that several noncooperative games have equilibria that are consistent with the same cooperative solution concept. For example, Fox and Bajari (2013) estimate a FCC spectrum auction matching model using pairwise stability while arguing that certain noncooperative dynamic games of private information in such ascending auctions have implicitly collusive Nash equilibria that satisfy pairwise stability.

B. *Matching Pairwise Stability versus Networks Pairwise Stability*

The definition of pairwise stability used in the matching literature (including matching with trading networks) is stronger than the most common definition in the related networks literature (e.g., Jackson and Wolinsky 1996). The *matching definition of pairwise stability* says there exists no vector of prices for trades $(p_\omega)_{\omega \in \Omega}$ such that two agents would prefer to make a trade ω *instead of* or in addition to their current trades and no agents would prefer to drop trades while the *networks or "Jackson" definition of pairwise stability* says only that no two agents would prefer to add or drop trades. There is no swapping of trades considered under the networks definition.

Any pairwise stable market outcome under the matching definition is pairwise stable under the weaker networks definition. The networks definition, while weaker and hence more robust, can thus lead to additional pairwise stable market outcomes and hence a wider identified set for the parameters of interest if the researcher is agnostic about equilibrium selection (e.g., Ciliberto and Tamer 2009). Nonparametric identification of valuation functions has been studied for the matching definition but not the weaker networks definition (Fox 2010; Graham 2011).

C. *Pairwise Stability versus Competitive Equilibrium*

A market outcome that is pairwise stable is robust to deviations by two agents at a time while a market outcome that is *groupwise stable* is robust to deviations by groups of arbitrary size. One can prove that a groupwise stable market outcome is also a *competitive equilibrium* and that competitive equilibria are efficient in the sense of maximizing economywide output (Hatfield et al. 2013).[1] This distinction is important in Fox and Bajari (2013), who impose pairwise stability but not groupwise stability, and hence argue that they can use their structural estimates of valuations to measure inefficiency in a spectrum auction. The trades in a competitive equilibrium are unique if the set of trades that maximizes economywide output is unique.[2]

D. *Solution Concept Existence*

Some researchers have estimated matching games without a proof of the existence of the imposed solution concept. For example, consider a matching game with a finite number of agents where there exists only one agent who can make multiple trades as a seller and that agent has complementarities across the multiple trades. Then there exist preferences for the other agents where no competitive equilibrium exists (Hatfield et al. 2013). On the other hand, an equilibrium exists if there is instead a continuum of agents in the market or agents can make divisible investments in consummated trades (Azevedo and Hatfield 2015; Hatfield and Kominers 2015). Imposing solution concepts that might not exist is dangerous ground but working with a continuum of agents is one way of precisely stating an approximation that is needed to ensure existence.

[1] A competitive equilibrium is a price vector p^{Ω} such that all agents pick trades to maximize profits and the market clears: supply equals demand on each trade.

[2] If preferences satisfy a substitutes condition, all pairwise stable markets outcomes are also groupwise stable and competitive equilibria (Hatfield et al. 2013). Many empirical applications focus on complementarities.

E. *Continuum versus Truly Finite Markets*

A *truly finite market* is when the set of agents in the data is equal to the set of agents in the structural matching game for a market, the set *I*. A *continuum market* is when the set of agents in the data is a finite subset of a continuum of agents in the matching game, *I*. In either case, a researcher might model one or multiple markets and explore various asymptotic arguments for frequentist estimation and inference.

Researchers working with marriage datasets or data from one large market have typically assumed that the true market is a continuum (e.g., Choo and Siow 2006; Fox and Bajari 2013). Assumptions on a continuum market are typically made such that the aggregate market outcome is a function of the distribution of $\left(\epsilon^{i}_{\Phi,\Psi}\right)_{\Phi\subseteq\Omega,\Psi\subseteq\Omega}$ across agents i but not particular realizations of the unobservables. Therefore, the market outcome is econometrically deterministic in the aggregate. By focusing on the equilibrium played in the data, there is no need to specify an equilibrium selection rule.

Researchers in other settings view each market as truly finite and allow the market outcome in each market to be a function of the realizations of $\left(\epsilon^{i}_{\Phi,\Psi}\right)^{i\in I}_{\Phi\subseteq\Omega,\Psi\subseteq\Omega}$. In the truly finite market, multiple equilibria typically lead to set identification if the researcher is completely agnostic about the selection rule over multiple values of a solution concept (e.g., Ciliberto and Tamer 2009). Fox (2010) discusses a "medium" assumption on the equilibrium selection rule that maintains point identification.

F. *Data on Prices of Trades*

In all papers on structurally estimating matching games, a researcher has data on aspects of the trades $\left(\Phi_i,\Psi_i\right)_{i\in I}$. A few matching papers have explored estimation also using data on the prices of trades $\left(p_{\omega}\right)_{\omega\in\Omega}$ (e.g., Akkus, Cookson, and Hortaçsu 2016; Fox and Bajari 2013; Uetake and Watanabe 2016). Assumptions can be imposed on the distribution of $\left(\epsilon^{i}_{\Phi,\Psi}\right)^{i\in I}_{\Phi\subseteq\Omega,\Psi\subseteq\Omega}$ such that the prices of trades $\left(p_{\omega}\right)_{\omega\in\Omega}$ are not functions of the realizations of $\left(\epsilon^{i}_{\Phi,\Psi}\right)^{i\in I}_{\Phi\subseteq\Omega,\Psi\subseteq\Omega}$. This simplifies both identification arguments and the computation of estimators. The related hedonics literature focuses more on data on the prices of trades (e.g., Heckman, Matzkin, and Nesheim 2010).

An econometric selection problem may arise if the prices of trades are jointly distributed with $\left(\epsilon^{i}_{\Phi,\Psi}\right)^{i\in I}_{\Phi\subseteq\Omega,\,\Psi\subseteq\Omega}$ and the prices of trades are observed only for consummated trades. Typically, the so-called generalized Roy selection model leads to issues of partial identification and slow rates of convergence. Similar selection issues may arise if the researcher has data on say the *profits* or *valuations* of consummated trades only.

G. *Quotas*

Define a *quota* q_i to be the maximum number of trades an agent i can undertake; valuations are $-\infty$ if the agent makes more trades. In marriage, the quota of each agent is known to be 1 (Choo and Siow 2006). In the matching of suppliers of car parts to the assemblers of cars, the quota of each assembler is the number of different car parts needed from an engineering perspective but the quota of each supplier is unknown (Fox forthcoming). To compute market outcomes to a matching game within an estimator, one needs to specify quotas or integrate them out over some distribution. The matching maximum score estimator of Fox (forthcoming) bases estimation off of inequalities that may remain valid in the presence of missing data on quotas.

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